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Yang-Mills—Einstein supergravity in seven dimensions

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We construct couplings of n vector multiplets to seven-dimensional $N=2$ supergravity. The $3n$ scalars of the theory parametrize the coset $SO(n,3)/SO(n)\times SO(3)$. The $(n+3)$ vector fields are used to gauge either an $SO(3)\times H$ (dimension $H=n$) or an $SO(3,1)\times H$ (dim $H=n-3$) subgroup of $SO(n,3)$. The theory has an indefinite potential which triggers compactification into $(\text{Minkowski})_4\times S^3$, with surviving $N=1$ supersymmetry.

I. INTRODUCTION

Pure supergravity theories in higher dimensions seem to have difficulties in yielding $d=4$ supergravity theories with large enough symmetries and a realistic particle spectrum. On the other hand, Yang-Mills-coupled higher-dimensional supergravities may circumvent these difficulties, since in such theories the Yang-Mills sector may provide the unifying symmetries.[1] Furthermore, it has been emphasized by Günaydin *et al.*[2] that Yang-Mills-coupled supergravity theories can be truly unified theories in the sense that there may exist a (noncompact) symmetry which relates all the fields of the theory to each other. Another advantage of these theories is that they could provide a super-Higgs mechanism through a suitable compactification which may not be possible in (conventional) four-dimensional supergravities.[3,4]

Some of the features mentioned above, if not all, have been realized in various matter-coupled higher-dimensional supergravities.[2,5–11] In this paper, we shed further light on the structure of such theories by constructing the couplings of an arbitrary number of vector multiplets to $N=2$ supergravity in seven dimensions. This theory is particularly interesting in that, as we shall show later, it admits $(\text{Minkowski})_4\times S^3$ compactification with an $N=1$ remaining supersymmetry.

Our results can be summarized as follows. We find that the $3n$ scalars of the n vector multiplets parametrize the manifold $SO(n,3)/SO(n)\times SO(3)$. All the couplings of the scalar fields are described in terms of a single function of scalars which is nothing but the coset representative of the scalar manifold. We then gauge an $(n+3)$-parameter subgroup of $SO(n,3)$ which uses all the vectors of the theory. An important consequence of this gauging is the emergence of a potential which has the form

$$V(\phi)=\tfrac{1}{4}e^{-\sigma}(C^{ai}{}_{j}C_{ai}{}^{j}-\tfrac{1}{9}C^2)\ .$$

Here $C_{ai}{}^{j}$ and C are functions of the scalars, which depend on the structure constants of the gauge group [see Eq. (3.4) in the text]. Note that the potential is indefinite. Exploiting this property and identifying[1,3,12,13] the SO(3) sector of the Yang-Mills fields with the spin connection of an S^3, we find that the theory admits $(\text{Minkowski})_4\times S^3$ compactification in which $N=1$ supersymmetry survives.

II. CONSTRUCTION OF UNGAUGED MATTER COUPLINGS

In this section we will describe the coupling of the $N=2$, $d=7$ supergravity[9,14] to n Abelian vector multiplets. The gauged version of these couplings will be discussed in the next section.

Pure $N=2$ supergravity in $d=7$ consists of the following fields: a siebenbein $e_\mu{}^m$, Sp(1) pseudo-Majorana[15] gravitinos $\psi_\mu{}^i$ $(i=1,2)$,[16] a triplet of vector fields $A_{\mu j}{}^i$, an Sp(1) pseudo-Majorana spinor χ^i, an antisymmetric tensor field $B_{\mu\nu}$, and a real scalar σ. In the following, we will couple n Abelian vector multiplets to these supergravity fields. One such multiplet contains a vector field A_μ, an Sp(1) pseudo-Majorana spinor λ^i, and an Sp(1) triplet of scalar fields.

It is expected[2,7,8,11,17] that in the coupled system the scalars ϕ^α $(\alpha=1,\ldots,3n)$ of the n vector multiplets parametrize the coset $SO(n,3)/SO(n)\times SO(3)$. [The SO(3) of the isotropy group is identified with the Sp(1) automorphism group of the supersymmetry algebra.] The scalar fields ϕ^α are described by an $(n+3)\times(n+3)$ matrix $L_I{}^A(\phi^\alpha)$, which is a representative element of the scalar coset manifold. It is convenient to decompose the index A into a **3** of Sp(1) and an **n** of SO(n), i.e.,

$$L_I{}^A\rightarrow(L_{Ij}{}^{i},L_I^a)\ ,\quad \begin{array}{l} I,A=1,\ldots,n+3\ ,\\ i,j=1,2\ ,\\ a=1,\ldots,n\ .\end{array} \tag{2.1}$$

The orthogonality condition of the $SO(n,3)$ matrix element $L_I{}^A$ is $L^T\eta L=\eta$, where $\eta_{IJ}=\text{diag}(-,-,-,+,\ldots,+)$. In terms of the fields defined in (2.1), it reads

$$-L_{Ij}{}^{i}L_{Ji}{}^{j}+L_I^aL_J^a=\eta_{IJ}\ . \tag{2.2}$$

The inverse matrix $L^I{}_A$, which is given by $L^{-1}=\eta L^T\eta$, satisfies the relations

$$\begin{aligned} &L_I^aL^{Ii}{}_j=0\ ,\\ &L_{Ij}{}^{i}L^{Ik}{}_l=-\delta_l^i\delta_j^k+\tfrac{1}{2}\delta_j^i\delta_l^k\ ,\\ &L_I^aL_b^I=\delta_b^a\ . \end{aligned} \tag{2.3}$$

As usual, the matrix $L_I{}^A$ transforms under global $SO(n,3)$

from the left and local composite $SO(n)\times SO(3)$ from the right. In particular, the latter transformations are

$$\delta L_a^I = L_b^I \Lambda^b{}_a(\phi)\ ,$$
$$\delta L^{Ii}{}_j = \tfrac{1}{2} L^{Ik}{}_j \Lambda_k^i(\phi) - \tfrac{1}{2} L^{Ii}{}_k \Lambda_j^k(\phi)\ , \tag{2.4}$$

where $\Lambda_a{}^b = -\Lambda^b{}_a$ and $\Lambda_j{}^i = \Lambda^j{}_i$ are the parameters of the local composite $SO(n)$ and $SO(3)$ transformations.

Next we decompose the Maurer-Cartan form $L^{-1}dL$ as

$$P_{\mu a j}^{\ \ i} = L_a^I \partial_\mu L_{Ij}^{\ i}\ ,$$
$$Q_{\mu j}^{\ i} = L^{Ii}{}_k \partial_\mu L_{Ij}^{\ k}\ , \tag{2.5}$$
$$Q_{\mu ab} = L^I{}_a \partial_\mu L_{Ib}\ .$$

From Eqs. (2.4) and (2.5) it follows that

$$\delta Q_{\mu a}{}^b = \partial_\mu \Lambda_a{}^b - Q_{\mu c}{}^b \Lambda_a{}^c + Q_{\mu a}{}^c \Lambda_c{}^b\ ,$$
$$\delta Q_{\mu j}^{\ i} = \partial_\mu \Lambda_j^i - \tfrac{1}{2} Q_{\mu k}^{\ i} \Lambda_j^k + \tfrac{1}{2} Q_{\mu j}^{\ k} \Lambda_k^i\ . \tag{2.6}$$

Owing to these transformation rules, both P_μ and Q_μ play an important role in the construction of the scalar coupling to supergravity.[18] We have collected some identities for P_μ and Q_μ in the Appendix.

We now require that all the fields of the coupled system transform according to definite representations of the global $SO(n,3)$ and the local composite $SO(n)\times Sp(1)$ symmetries of the coset space. The $n+3$ vector fields $A_\mu{}^I$ $(I=1,\ldots,n+3)$ transform according to the defining representation of $SO(n,3)$. The gravitinos $\psi_\mu{}^i$ and the χ^i of the supergravity multiplet are doublets of $Sp(1)$, while the λ^{ai} are in $(\mathbf{n},2)$ of the composite $SO(n)\times Sp(1)$. The remaining fields of the supergravity multiplet are singlets.

Using the symmetries discussed above, one can immediately write an ansatz for the action and the transformation rules up to a number of constant coefficients. We fix most of the coefficients in the transformation rules by requiring closure of the supersymmetry algebra on the bosonic fields. We then determine all the remaining coefficients by the supersymmetry of the action. Thus we obtain the following Lagrangian:

$$\begin{aligned}
e^{-1}\mathscr{L} = {} & \tfrac{1}{2}R - \frac{i}{2}\bar\psi_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho - \tfrac{1}{4} e^{\sigma} a_{IJ} F^I_{\mu\nu} F^J_{\mu\nu} - \tfrac{1}{12} e^{2\sigma} G_{\mu\nu\rho} G_{\mu\nu\rho} - \tfrac{5}{2} i \bar\chi \not{D} \chi - \tfrac{5}{8}(\partial_\mu \sigma)^2 - \frac{i}{2}\bar\lambda^a \not{D}\lambda_a - \tfrac{1}{2} P^{a\,i}_{\mu j} P_{\mu a i}^{\ \ j} \\
& - \tfrac{5}{4} i \bar\chi \gamma^\mu \gamma^\nu \psi_\mu \partial_\nu \sigma - \frac{1}{\sqrt{2}} \bar\lambda^{ai} \gamma^\mu \gamma^\nu \psi_{\mu j} P_{\nu a i}^{\ \ j} \\
& + \frac{i}{24\sqrt{2}} e^{\sigma} G_{\mu\nu\rho} (\bar\psi^\lambda \gamma_{[\lambda} \gamma^{\mu\nu\rho} \gamma_{\tau]} \psi^\tau + 4\bar\psi_\lambda \gamma^{\mu\nu\rho} \gamma^\lambda \chi - 3\bar\chi\, \gamma^{\mu\nu\rho}\chi + \bar\lambda^a \gamma^{\mu\nu\rho} \lambda_a) \\
& + \frac{1}{4\sqrt{2}} e^{\sigma/2} F^I_{\mu\nu} (\bar\psi^{\lambda i} \gamma_{[\lambda} \gamma^{\mu\nu} \gamma_{\tau]} \psi^\tau_j L_{I i}^{\ j} - 2\bar\psi^{\lambda i} \gamma^{\mu\nu} \gamma_\lambda \chi_j + 3\bar\chi^i \gamma^{\mu\nu} \chi_j L_{I i}^{\ j} - i\sqrt{2}\bar\psi_\lambda \gamma^{\mu\nu} \gamma^\lambda \lambda_a L_I{}^a \\
& \qquad + 2i\sqrt{2}\,\bar\lambda_a \gamma^{\mu\nu} \chi L_I^a - \bar\lambda_a^i \gamma^{\mu\nu} \lambda_j^a L_I{}^j{}_i) + \text{quartic fermions}\ ,
\end{aligned} \tag{2.7}$$

where a_{IJ} is defined by

$$a_{IJ} = L_{Ij}^{\ i} L_{Ji}^{\ j} + L_I^a L_{Ja}\ . \tag{2.8}$$

The action which follows from this Lagrangian is invariant under the following supersymmetry transformations:

$$\begin{aligned}
& \delta e_\mu^m = i\bar\epsilon \gamma^m \psi_\mu\ , \\
& \delta\psi_\mu^i = 2D_\mu \epsilon^i - \frac{1}{60\sqrt{2}} e^{\sigma} (\gamma_\mu \gamma^{mnp} + 5\gamma^{mnp}\gamma_\mu) G_{mnp} \epsilon^i + \frac{i}{10\sqrt{2}} e^{\sigma/2} (3\gamma_\mu \gamma^{mn} - 5\gamma^{mn}\gamma_\mu) F^I_{mn} L_{Ij}^{\ i} \epsilon^j\ , \\
& \delta B_{\mu\nu} = i\sqrt{2} e^{-\sigma} (\bar\epsilon \gamma_{[\mu} \psi_{\nu]} + \bar\epsilon \gamma_{\mu\nu} \chi) - \sqrt{2} A^I_{[\mu} \delta A^J_{\nu]} \eta_{IJ}\ , \\
& \delta A^I_\mu = -\sqrt{2} e^{-\sigma/2} (\bar\epsilon^i \psi_{\mu j} + \bar\epsilon^i \gamma_\mu \chi_j) L^{Ij}{}_i + i e^{-\sigma/2} \bar\epsilon \gamma_\mu \lambda^a L^I_a\ , \\
& \delta L_I^a = \sqrt{2}\bar\epsilon^i \lambda_j^a L_{I i}^{\ j}\ , \\
& \delta L_{Ij}^{\ i} = \sqrt{2}\bar\epsilon^i \lambda_{aj} L_I^a - \text{trace}\ , \\
& \delta\chi^i = \frac{i}{5\sqrt{2}} e^{\sigma/2} \gamma^{mn} F^I_{mn} L_{Ij}^{\ i} \epsilon^j - \frac{1}{15\sqrt{2}} e^{\sigma} \gamma_{mnp} G_{mnp} \epsilon^i - \tfrac{1}{2}\not{\partial}\sigma \epsilon^i\ , \\
& \delta\lambda^{ai} = -\tfrac{1}{2} e^{\sigma/2} \gamma^{mn} F^I_{mn} L_I^a \epsilon^i - \sqrt{2} i \gamma^m P^{a\ i}_{mj} \epsilon^j\ , \\
& \delta\sigma = -2i\bar\epsilon\chi\ .
\end{aligned} \tag{2.9}$$

The transformations of ψ_μ, χ, and λ contain additional terms bilinear in the fermionic fields which we have not determined. When Sp(1) indices are omitted, a northwest-southeast contraction is understood, e.g., $\bar\epsilon\gamma^m\psi_\mu=\bar\epsilon^i\gamma^m\psi_{\mu i}$. In the above the covariant derivative of ϵ and the field strengths $F^I_{\mu\nu}$ and $G_{\mu\nu\rho}$ are defined in the following way:

$$
\begin{aligned}
D_\mu\epsilon^i&=\partial_\mu\epsilon^i-\tfrac{1}{2}Q_{\mu j}{}^i\epsilon^j+\tfrac{1}{4}\omega_\mu{}^{mn}(e)\gamma_{mn}\epsilon^i\ ,\\
F^I_{\mu\nu}&=2\partial_{[\mu}A^I_{\nu]}\ ,\\
G_{\mu\nu\rho}&=3(\partial_{[\mu}B_{\nu\rho]}-\frac{1}{\sqrt{2}}A^I_{[\mu}F^J_{\nu\rho]}\eta_{IJ})\ .
\end{aligned}
\tag{2.10}
$$

III. CONSTRUCTION OF THE GAUGED MATTER COUPLINGS

In this section we describe how the results of the previous section can be extended by gauging an appropriate subgroup G of SO(n,3). The structure constants of this subgroup, which is not necessarily simple, are denoted by $f_{IJ}{}^K$. Since the $n+3$ vector fields should transform according to the adjoint representation of the Yang-Mills group, we require that dim $G=n+3$.

Let us first define the gauge-covariant extension of the geometrical objects which were introduced in the previous section. Clearly the definition of P_μ and Q_μ given in (2.5) has to be replaced by[7]

$$
\begin{aligned}
\tilde P_{\mu a j}{}^i&=L^I_a(\partial_\mu\delta^K_I+f_{IJ}{}^KA^J_\mu)L_{Kj}{}^i\ ,\\
\tilde Q_{\mu j}{}^i&=L^{Ii}{}_k(\partial_\mu\delta^K_I+f_{IJ}{}^KA^J_\mu)L_{Kj}{}^k\ ,\\
\tilde Q_{\mu ab}&=L^I_a(\partial_\mu\delta^K_I+f_{IJ}{}^KA^J_\mu)L_{Kb}\ ,
\end{aligned}
\tag{3.1}
$$

where the gauge coupling constants are absorbed into $f_{IJ}{}^K$. Similarly, the definition given in (2.10) must be replaced by

$$
\begin{aligned}
\tilde D_\mu\epsilon^i&=\partial_\mu\epsilon^i-\tfrac{1}{2}\tilde Q_{\mu j}{}^i\epsilon^j+\tfrac{1}{4}\omega_\mu{}^{mn}(e)\gamma_{mn}\epsilon^i\ ,\\
\tilde F^I_{\mu\nu}&=2\partial_{[\mu}A^I_{\nu]}+f_{KL}{}^IA^K_\mu A^L_\nu\ ,\\
\tilde G_{\mu\nu\rho}&=3\left[\partial_{[\mu}B_{\nu\rho]}-\frac{1}{\sqrt{2}}A^I_{[\mu}F^J_{\nu\rho]}\eta_{IJ}\right.\\
&\qquad\left.+\frac{1}{3\sqrt{2}}f_{IJ}{}^KA^I_\mu A^J_\nu A_{\rho K}\right]\ .
\end{aligned}
\tag{3.2}
$$

From (3.1) we see that in order to ensure the composite local SO(n)×SO(3) transformation properties of $\tilde P$ and $\tilde Q$ analogous to those given in (2.4), the structure constants must satisfy[7]

$$
f_{IJ}{}^L\eta_{KL}=f_{[IJ}{}^L\eta_{K]L}\ .
\tag{3.3}
$$

One way of satisfying this condition is to take η_{IJ} to be the Cartan-Killing metric of the gauged algebra. In that case, the group can be either SO(3)×H with dim $H=n$, or SO(3,1)×H with dim $H=(n-3)$. Another possibility is to take $\eta_{IJ}=(\eta_{mn},\delta_{AB})$ where $m=1,\dots,p$, $A=p+1,\dots,n+3$ and interpret η_{mn} as the Cartan-Killing metric of SO(3)×H [dim $H=(p-3)$], or SO(3,1)×H [dim $H=(p-6)$]. In both cases, there are $(n+3-p)$ U(1) factors in the gauge group.

It is convenient to introduce[7] the following projections of the structure constants $f_{IJ}{}^K$:

$$
\begin{aligned}
C&=if_{IJ}{}^KL^{Ii}{}_kL^{Jj}{}_iL_{Kj}{}^k\ ,\\
C^{ai}{}_j&=if_{IJ}{}^KL^{Ii}{}_kL^{Jk}{}_jL^a_K\ ,\\
C_{abj}{}^i&=f_{IJ}{}^KL^I_aL^J_bL_{Kj}{}^i\ .
\end{aligned}
\tag{3.4}
$$

Some of their properties are given in the Appendix.

To obtain the action and supersymmetry transformations of the gauged system one must first replace everywhere $G_{\mu\nu\rho}$, $F^I_{\mu\nu}$, P_μ, and Q_μ by $\tilde G_{\mu\nu\rho}$, $\tilde F^I_{\mu\nu}$, $\tilde P_\mu$, and $\tilde Q_\mu$, respectively. This procedure clearly breaks supersymmetry. To restore the supersymmetry of the action, the following modifications are needed in the Lagrangian:

$$
\begin{aligned}
e^{-1}\mathcal{L}_g
&=\frac{i\sqrt{2}}{24}e^{-\sigma/2}C(-\bar\psi_\mu\gamma^{\mu\nu}\psi_\nu-2\bar\psi_\mu\gamma^\mu\chi-3\bar\chi\chi+\bar\lambda^a\lambda_a)\\
&\quad+\tfrac{1}{2}e^{-\sigma/2}C_{aj}{}^i(\bar\psi^j_\mu\gamma^\mu\lambda^a_i-2\bar\chi^j\lambda^a_i)\\
&\quad+\frac{1}{\sqrt{2}}e^{-\sigma/2}C_{abj}{}^i\bar\lambda^{aj}\lambda^b_i-\tfrac{1}{4}e^{-\sigma}(C^{ai}{}_jC_{ai}{}^j-\tfrac{1}{9}C^2)\ .
\end{aligned}
\tag{3.5}
$$

In addition, the supersymmetry transformation rules must be modified by adding the g-dependent terms

$$
\begin{aligned}
\delta_g\psi^i_\mu&=-\frac{\sqrt{2}}{30}C\gamma_\mu\epsilon^ie^{-\sigma/2}\ ,\\
\delta_g\chi^i&=\frac{\sqrt{2}}{30}C\epsilon^ie^{-\sigma/2}\ ,\\
\delta_g\lambda^{ai}&=-iC^{aij}\epsilon_je^{-\sigma/2}\ .
\end{aligned}
\tag{3.6}
$$

IV. COMPACTIFICATION

In this section we investigate the compactification of our model into four-dimensional space-time.

First we determine the equations of motion. The ones which are relevant for compactification are given by

$$
\begin{aligned}
R'_{\mu\nu}&=e^\sigma a_{IJ}\tilde F^I_{\mu\rho}\tilde F_\nu{}^{J\rho}+\tfrac{1}{2}e^{2\sigma}\tilde G_{\mu\rho\sigma}\tilde G_\nu{}^{\rho\sigma}+\tfrac{5}{4}\partial_\mu\sigma\partial_\nu\sigma+\tilde P^{ai}_{\mu j}\tilde P^{aj}_{\nu i}-\tfrac{1}{2}g_{\mu\nu}\Box\sigma\ ,\\
\Box\sigma&=\tfrac{1}{5}e^\sigma a_{IJ}\tilde F^I_{\mu\nu}\tilde F^{\mu\nu J}+\tfrac{2}{15}e^{2\sigma}\tilde G_{\mu\nu\rho}\tilde G^{\mu\nu\rho}-\tfrac{1}{5}e^{-\sigma}(C^{ai}{}_jC^{aj}{}_i-\tfrac{1}{9}C^2)\ ,\\
D_\mu\tilde P^{\mu ai}{}_j&=e^\sigma F^I_{\mu\nu}F^{\mu\nu J}L^a_IL_{Jj}{}^i-ie^{-\sigma}C^{abi}{}_kC_{bj}{}^k-\text{trace}\ ,\\
\tilde D_\mu(a_{IJ}e^\sigma\tilde F^{\mu\nu J})&=-\frac{e^{2\sigma}}{\sqrt{2}}\tilde G^{\nu\rho\sigma}\tilde F^J_{\rho\sigma}\eta_{IJ}+\tilde P^{\nu ai}{}_j(L^b_IC_{abi}{}^j-\frac{i}{2}C_{ak}{}^jL_{Ii}{}^k)\ ,\\
D_\mu(e^{2\sigma}\tilde G^{\mu\nu\rho})&=0\ .
\end{aligned}
\tag{4.1}
$$

Note that as in six dimensions[3,7] the trace of the Einstein equation is proportional to the field equation of the scalar σ, which is crucial for compactification to Minkowski space-time. Note also the presence of the negative definite term $C_{aj}{}^{i}C_{ai}{}^{j}$ in the σ scalar equation of motion, which is needed for solving this equation.

In this paper we shall concentrate on compactification of the theory into $(\text{Minkowski})_4\times S^3$. We first take the Yang-Mills group to be $SO(3,1)\times H$ where $\dim H=(n-3)$ [we will discuss the case of $SO(3)\times H$ at the end of this section]. We then consider the following ansatz:

$$g_{MN}dx^M dx^N=\eta_{\mu\nu}dx^\mu dx^\nu+g_{\underline{\mu}\underline{\nu}}(S^3)dx^{\underline{\mu}}dx^{\underline{\nu}}\,, \tag{4.2a}$$

$$F^{\underline{c}}_{\underline{\mu}\underline{\nu}}=\pm g'\epsilon_{\underline{\mu}\underline{\nu}\underline{\lambda}}e^{\underline{\lambda}\underline{c}}\,, \tag{4.2b}$$

$$L^{ni}{}_{j}=\frac{1}{\sqrt{2}}(\tau^n)^i_j\,, \tag{4.2c}$$

$$L^{\underline{c}_1}_{\underline{c}_2}=\delta^{\underline{c}_1}_{\underline{c}_2}\,, \tag{4.2d}$$

with all the other fields vanishing in the background. Here, $\eta_{\mu\nu}$ is the $(\text{Minkowski})_4$ metric, $g_{\underline{\mu}\underline{\nu}}$ $(\underline{\mu}=5,6,7)$ is the metric on S^3, g' is the SO(3,1) coupling constant, and $e_{\underline{\mu}}{}^{\underline{c}}$ is the dreibein on S^3. Moreover, c and n label the three compact and three noncompact directions in the SO(3,1) algebra, respectively, and τ^n are the usual Pauli matrices. Note that (4.2b) implies that the SO(3) Yang-Mills field is identified with the SO(4)-invariant spin connection of the three-sphere. Hence, from (4.2) it is seen that our background has $SO(4)\times H$ [$\dim H=(n-3)$] invariance. From (4.2) and (3.4) it follows that

$$C=C_{abj}{}^{i}=0,\quad C_{\underline{a}j}{}^{i}=g'(\tau_{\underline{a}})^i_j \quad \text{(zero otherwise)}\,, \tag{4.3a}$$

$$R_{\underline{\mu}\underline{\nu}}{}^{\underline{ab}}=g'^2(e^{\underline{a}}_{\underline{\mu}}e^{\underline{b}}_{\underline{\nu}}-e^{\underline{a}}_{\underline{\nu}}e^{\underline{b}}_{\underline{\mu}})\,. \tag{4.3b}$$

Using these relations, one easily verifies that the background given in (4.2) indeed satisfies all the field equations. The sign ambiguity in Eq. (4.2b) will be resolved below by the requirement of the supersymmetry of the background.

If one takes $SO(3)\times H$ ($\dim H=n$) as a Yang-Mills group, then in the background given by (4.2) [where g' now denotes the SO(3) coupling constant] $C\neq 0$ and $C_{aj}{}^{i}=0$. Therefore, the scalar field equation cannot be satisfied. Hence we will not consider this case further in this paper.

In order to determine the supersymmetry of the background one must examine the supersymmetry transformations of the fermionic fields. In our background the only relevant ones are $\delta\psi_\mu$ and $\delta\lambda^a$. The vanishing of the latter requires

$$\sigma_m(\tau^m)^i_j\epsilon^j=\pm 3\epsilon^i\,. \tag{4.4}$$

Here we have used the following conventions for the $d=7$ γ matrices:

$$\gamma_\mu=\tilde{\gamma}_\mu\otimes\mathbb{1},\quad \gamma_{\underline{\mu}}=\tilde{\gamma}_5\otimes\sigma_{\underline{\mu}}\,, \tag{4.5}$$

where $\tilde{\gamma}_\mu$ and $\sigma_{\underline{\mu}}$ are the γ matrices in four and three dimensions, respectively. The 4×4 matrix $\sigma^m\otimes\tau^m$ has three eigenvalues ϵ_p $(p=1,2,3)$ which satisfy $\sigma^m\otimes\tau^m\epsilon_p=\epsilon_p$, and one eigenvalue ϵ_4 which satisfies[13] $\sigma^m\otimes\tau^m\epsilon_4=-3\epsilon_4$. Therefore from (4.4) we see that the upper sign corresponds to no supersymmetry, while the lower sign implies $N=1$ supersymmetry of the background. It is easy to verify that ϵ_4 also satisfies $\delta\psi_\mu=0$.

V. DISCUSSION

To summarize our results in this paper we have constructed the couplings of an arbitrary number of Yang-Mills multiplets to $d=7$, $N=2$ supergravity, with Yang-Mills group $SO(3)\times H$ or $SO(3,1)\times H$. Furthermore, we have shown that in the latter case the theory compactifies into $(\text{Minkowski})_4\times S^3$ with $N=1$ surviving supersymmetry. We recall that the choice for the gauge group is dictated by the condition given in Eq. (3.3). It would be interesting to see whether this condition can be relaxed. Another question, which is seemingly related to this one, is whether the full Yang-Mills group can be simple.

We would like to point out that our gauging procedure is apparently different from the one described in Ref. 2, since the analog of our Eq. (3.2) does not exist in the case of (minimal) U(1) gauging in five dimensions.

In coupling n vector multiplets to $N=4$, $d=4$ supergravity, one encounters structures similar to those described above. In that case, the scalar manifold is $[SO(n,6)/SO(n)\times SO(6)]\times[SU(1,1)/U(1)]$. The ungauged couplings of this manifold were constructed by de Roo.[17] Applying the same techniques which we have used above, we have gauged[19] an $(n+6)$-dimensional subgroup of $SO(n,6)$ which is either $SO(4)\times H$ ($\dim H=n$) or $SO(4,1)\times H$ ($\dim H=n-4$). In particular, we have found that the potential is given by

$$V(z,\phi)=\frac{(1-z)(1-z^*)}{(1-zz^*)}(C^{ai}{}_{j}C_{ai}{}^{j}-\tfrac{4}{9}C^{ij}C_{ij})\,, \tag{5.1}$$

where z is the complex scalar of the $N=4$ supergravity multiplet, which parametrizes the SU(1,1)/U(1) coset and $C_{aj}{}^{i}=C_{aj}{}^{i}(C_{ai}{}^{i}=0,\ i=1,\ldots,4)$ and $C^{ij}=C^{ji}$ are the analog of the $C_{aj}{}^{i}$ and C defined in Eq. (3.4). It would be interesting to examine the minima of this scalar potential and the supersymmetry of these minima, to see whether it can lead to realistic super-Higgs effect. One may speculate that such minima may eventually be related to a certain compactification of the anomaly-free[20] $N=1$, $d=10$ supergravity theory.

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APPENDIX

In this appendix we give some relevant identities for $\tilde{P}_\mu$, $\tilde{Q}_\mu$ and the C functions, which are defined in Eqs. (3.1) and (3.4), respectively. Corresponding identities for P_μ and Q_μ [see Eq. (2.5)] are easily obtained by setting the structure constants $f_{IJ}{}^{K}$ equal to zero.

The P_μ and Q_μ satisfy the Maurer-Cartan equations which follow from the observation that $L^{-1}(d+A)L$ has the form of a trivial gauge field whose curvature vanishes. In the present case these equations read

$$D_{[\mu}\widetilde{P}_{\nu]}{}^{ai}{}_{j}=\tfrac{1}{2}\widetilde{F}^{I}_{\mu\nu}(A)L_{Ib}C^{abi}{}_{j} -\tfrac{1}{2}i[\widetilde{F}^{I}_{\mu\nu}(A)L_{I}{}^{k}{}_{j}C^{ai}{}_{k}-\text{trace}] \,, \qquad \text{(A1)}$$

$$\widetilde{P}_{[\mu}{}^{k}{}_{j}\widetilde{P}_{\nu]}{}^{i}{}_{k}=-\tfrac{1}{2}\widetilde{Q}_{\mu\nu}{}^{i}{}_{j}+\frac{i}{6}C\widetilde{F}^{I}_{\mu\nu}(A)L_{I}{}^{i}{}_{j}+\frac{i}{2}C^{ai}{}_{j}\widetilde{F}^{I}_{\mu\nu}(A)L_{I}{}^{a} \,,$$

where $\widetilde{F}^{I}_{\mu\nu}(A)$ and $\widetilde{Q}_{\mu\nu}{}^{i}{}_{j}$ are the field strengths of A^{I}_{μ} and $\widetilde{Q}_{\mu}{}^{i}{}_{j}$, respectively.

Derivatives of the C functions yield again C functions. For instance,

$$\begin{aligned} D_{\mu}C^{ai}{}_{j}&=-\tfrac{1}{3}C\widetilde{P}_{\mu}{}^{ai}{}_{j}+2i(C^{abk}{}_{j}\widetilde{P}_{\mu bk}{}^{i}-\text{trace}) \,, \\ D_{\mu}C&=-3C^{ai}{}_{j}\widetilde{P}_{\mu ai}{}^{j} \,. \end{aligned} \qquad \text{(A2)}$$

Similarly the derivative of a representative element L gives a P_{μ},

$$\begin{aligned} \widetilde{D}_{\mu}L_{I}{}^{i}{}_{j}&=\widetilde{P}_{\mu}{}^{ai}{}_{j}L_{Ia} \,, \\ \widetilde{D}_{\mu}L_{I}{}^{a}&=\widetilde{P}_{\mu}{}^{ai}{}_{j}L_{I}{}^{j}{}_{i} \,. \end{aligned} \qquad \text{(A3)}$$

In products of C functions one can make use of the Jacobi identity $f_{[IJ}{}^{L}f_{K]LM}=0$. This leads, for instance, to the following identity:

$$C_{ab}{}^{ij}C_{bij}=0 \,. \qquad \text{(A4)}$$

We have used this relation in proving the supersymmetry of the action.

We finally give the transformation properties of the C functions. It is convenient to define the following projections of the variations of the scalars:

$$\begin{aligned} \delta L_{I}{}^{i}{}_{j}&=X_{a}{}^{i}{}_{j}L_{I}{}^{a}+(X^{i}{}_{k}L_{I}{}^{k}{}_{j}-\text{trace}) \,, \\ \delta L_{I}{}^{a}&=X^{ab}L_{Ib}+X^{ai}{}_{j}L_{I}{}^{j}{}_{i} \,. \end{aligned} \qquad \text{(A5)}$$

Using these definitions one can easily show that

$$\begin{aligned} \delta C^{2}&=-6CC^{ai}{}_{j}X_{a}{}^{j}{}_{i} \,, \\ \delta(C^{ai}{}_{j}C_{a}{}^{j}{}_{i})&=4iC_{a}{}^{i}{}_{j}X_{b}{}^{j}{}_{k}C^{abk}{}_{i}-\tfrac{2}{3}C^{ai}{}_{j}X_{a}{}^{j}{}_{i}C \,. \end{aligned} \qquad \text{(A6)}$$

Note that due to these relations the terms proportional to $C_{a}{}^{i}{}_{j}C$ cancel in the variations of our potential.

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[16]In the following it is understood that Sp(1) indices $i, j, \ldots$ are contracted with the invariant antisymmetric tensor ϵ_{ij} ($\epsilon_{12}=\epsilon^{12}=+1$), i.e., $A^{i}B_{i}=-A_{i}B^{i}=\epsilon^{ji}A_{i}B_{j}$. Sp(1) pseudo-Majorana spinors satisfy $\chi^{i}=C\epsilon^{ij}\overline{\chi}_{j}{}^{T}$, $C^{T}=C$, $\overline{\chi}^{i}\gamma^{\nu_1\cdots\nu_n}\lambda^{j}=(-)^{n+1}\overline{\lambda}^{j}\gamma^{\nu_n\cdots\nu_1}\chi^{i}$.

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